

Tracing the Interplay between Non-Thermal Dark Matter and Right-Handed Dirac Neutrinos with LHC Data

LUIS A. ANCHORDOQUI¹, HAIM GOLDBERG², BRIAN VLCEK¹¹ Department of Physics, University of Wisconsin-Milwaukee, Milwaukee, WI 53201, USA² Department of Physics, Northeastern University, Boston, MA 02115, USA

brian.vlcek@gmail.com

Abstract: Light-element abundances probing big bang nucleosynthesis (BBN) and precision data from cosmology probing the cosmic microwave background (CMB) decoupling epoch have hinted at the presence of extra relativistic degrees of freedom. This is widely referred to as “dark radiation”, suggesting the need for new light states in the UV completion of the standard model (SM). We provide a brief and concise overview of the current observational status of such dark radiation and investigate the interplay between two possible interpretations of the extra light states: the right-handed partners of three Dirac neutrinos (which interact with all fermions through the exchange of a new heavy vector meson Z') and dark matter (DM) particles that were produced through a non-thermal mechanism, such as late time decays of massive relics. This model ties together cosmological indications of extra light states and the production of the heavy vector particle at the Large Hadron Collider (LHC).

Keywords: dark radiation, dark matter, right-handed Dirac neutrinos.

To accommodate new physics in the form of extra relativistic degrees of freedom (r.d.o.f.) it is convenient to account for the extra contribution to the SM energy density, by normalizing it to that of an “equivalent” neutrino species. The number of “equivalent” light neutrino species, $N_{\text{eff}} \equiv (\rho_R - \rho_\gamma)/\rho_{\nu_L}$, quantifies the total “dark” relativistic energy density (including the three left-handed SM neutrinos) in units of the density of a single Weyl neutrino: $\rho_{\nu_L} = (7\pi^2/120)(4/11)^{4/3}T_\gamma^4$, where ρ_γ is the energy density of photons (with temperature T_γ) and ρ_R is the total energy density in relativistic particles [1]. A selection of the most recent measurements of N_{eff} together with the 1σ confidence intervals from various combinations of models and data sets are shown in Fig. 1. The data hint at the presence of an excess ΔN above SM expectation of $N_{\text{eff}} = 3.046$ [2].

One of the most striking results of the Planck spacecraft is that the best-fit Hubble constant has the value $h = 0.674 \pm 0.012$ [3].¹ This result deviates by more than 2σ from the value obtained with the Hubble Space Telescope, $h = 0.738 \pm 0.024$ [4]. The impact of the new h determination is particularly complex in the investigation of N_{eff} . Combining CMB observations with data from baryon acoustic oscillations (BAO) the Planck Collaboration reported $N_{\text{eff}} = 3.30 \pm 0.27$. Adding the H_0 measurement to the CMB data gives $N_{\text{eff}} = 3.62 \pm 0.25$ and relieves the tension between the CMB data and H_0 at the expense of new neutrino-like physics (at around the 2.3σ level). We have nothing further to add to this discussion, but it seems worthwhile to analyze any other independent set of data [5].

Several models have been proposed to explain ΔN . To accommodate recent Planck observations in this brief communication we re-examine two of them: models based on milli-weak interactions of right-handed partners of three Dirac neutrinos [6], and models in which the extra relativistic degrees of freedom are related to possible dark matter (DM) candidates produced via decay of heavy relics [7].² An interesting consequence of such a non-thermal DM scenario is that if the lifetime of the decaying particle is longer than about 10^3 seconds, the expansion history of the uni-

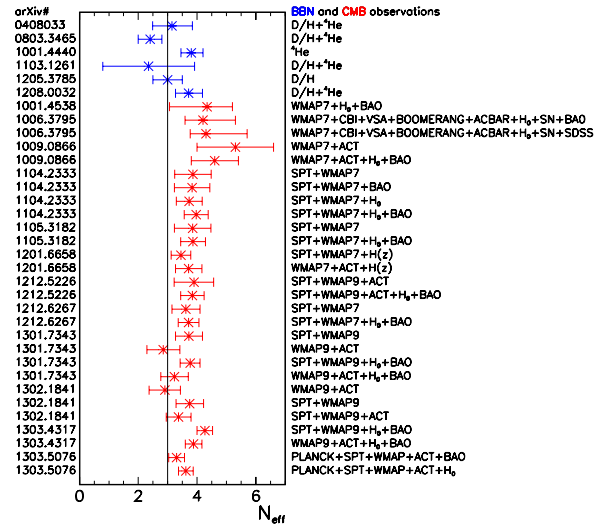


Fig. 1: A selection of the most recent cosmological N_{eff} measurements and the 1σ confidence intervals from various combinations of models and data sets. BBN findings are shown in blue and those from the CMB epoch in red.

verse during the era of BBN will not have DM contributions to N_{eff} . Moreover, if there is a light DM particle that annihilates to photons after the ν_L have decoupled, the photons are heated beyond their usual heating from e^\pm annihilation, reducing the late time ratio of neutrino and photon temperatures (and number densities), leading to $N_{\text{eff}} < 3$ [8]. This opens the window for addition of one or more ν_R neutrino flavors and still be consistent with $N_{\text{eff}} = 3$.

1. We adopt the usual convention of writing the Hubble constant at the present day as $H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$. For $t = \text{today}$, the various energy densities are expressed in units of the critical density ρ_c ; e.g., the matter density $\Omega_M \equiv \rho_M(\text{today})/\rho_c$.
2. ν_R 's have long been suspected to contribute to N_{eff} [9].

We begin by first establishing the contribution of right-handed neutrinos to N_{eff} , that is ΔN_ν as a function of the ν_R decoupling temperature. A critical input for ΔN_ν is the relation between the r.d.o.f. and the temperature of the primordial plasma. This relation is complicated because the temperature which is of interest for right-handed neutrino decoupling from the heat bath may lie in the vicinity of the quark-hadron cross-over transition. To connect the temperature to an effective number of r.d.o.f. we make use of some high statistics lattice simulations of a QCD plasma in the hot phase, especially the behavior of the entropy during the changeover [10].

Taking into account the isentropic heating of the rest of the plasma between $T_{\nu_R}^{\text{dec}}$ and $T_{\nu_L}^{\text{dec}}$ decoupling temperatures we obtain $\Delta N_\nu = 3[g_s(T_{\nu_L}^{\text{dec}})/g_s(T_{\nu_R}^{\text{dec}})]^{4/3}$, where $g_s(T)$ is the effective number of interacting (thermally coupled) r.d.o.f. at temperature T ; *e.g.*, $g_s(T_{\nu_L}^{\text{dec}}) = 43/4$. At energies above the deconfinement transition towards the quark gluon plasma, quarks and gluons are the relevant fields for the QCD sector, such that the total number of SM r.d.o.f. is $g_s = 61.75$. As the universe cools down, the SM plasma transitions to a regime where mesons and baryons are the pertinent degrees of freedom. Precisely, the relevant hadrons present in this energy regime are pions and charged kaons, such that $g_s = 19.25$ [11]. This significant reduction in the degrees of freedom results from the rapid annihilation or decay of any more massive hadrons which may have formed during the transition. The quark-hadron crossover transition therefore corresponds to a large redistribution of entropy into the remaining degrees of freedom. Concretely, the effective number of interacting r.d.o.f. in the plasma at temperature T is given by $g_s(T) \simeq r(T) (N_B + \frac{7}{8} N_F)$, with $N_B = 2$ for each real vector field and $N_F = 2$ for each spin- $\frac{1}{2}$ Weyl field. The coefficient $r(T)$ is unity for leptons, two for photon contributions, and is the ratio $s(T)/s_{\text{SB}}$ is the actual (ideal Stefan-Boltzmann) entropy shown in Fig 2. The entropy rise during the confinement-deconfinement changeover can be parametrized, for $150 \text{ MeV} < T < 500 \text{ MeV}$, by

$$\frac{s}{T^3} \simeq \frac{42.82}{\sqrt{392\pi}} e^{-\frac{(T_{\text{MeV}}-151)^2}{392}} + \left(\frac{195.1}{T_{\text{MeV}} - 134} \right)^2 \times 18.62 \frac{e^{195.1/(T_{\text{MeV}}-134)}}{[e^{195.1/(T_{\text{MeV}}-134)} - 1]^2}. \quad (1)$$

For the same energy range, we obtain

$$g_s(T) \simeq 47.5 r(T) + 19.25. \quad (2)$$

In Fig. 2 we show $g_s(T)$ as given by (2). Our parametrization is in very good agreement with the phenomenological estimate of [12].

If relativistic particles are present that have decoupled from the photons, it is necessary to distinguish between two kinds of r.d.o.f.: those associated with the total energy density g_ρ , and those associated with the total entropy density g_s . Since the quark-gluon energy density in the plasma has a similar T dependence to that of the entropy (see Fig. 7 in [10]), we take $g_\rho(T) \simeq r(T) (N_B + \frac{7}{8} N_F)$.

The right-handed neutrino decouples from the plasma when its mean free path becomes greater than the Hubble radius at that time

$$\Gamma(T_{\nu_R}^{\text{dec}}) = H(T_{\nu_R}^{\text{dec}}), \quad (3)$$

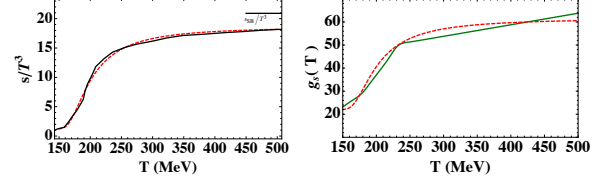


Fig. 2: Left: The parametrization of the entropy density given in Eq. (1) (dashed line) superposed on the result from high statistics lattice simulations [10] (solid line). **Right:** Comparison of $g_s(T)$ obtained using Eq. (2) (dashed line) and the phenomenological estimate of [12] (solid line).

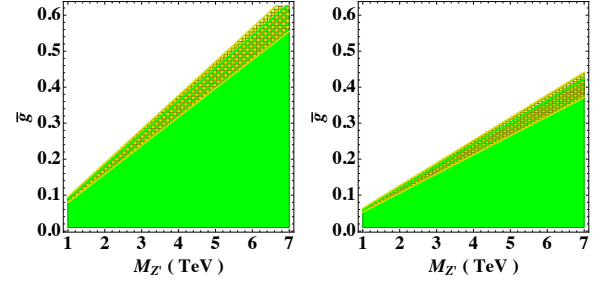


Fig. 3: The (green) shaded area shows the 1σ region allowed from decoupling requirements to accommodate Planck + BAO, whereas the (yellow) cross-hatched area shows the region allowed from decoupling requirements to accommodate Planck + H_0 . We have taken $\mathcal{K} = 0.5$ (left) and $\mathcal{K} = 2.5$ (right).

where

$$\Gamma(T_{\nu_R}^{\text{dec}}) = \mathcal{K} \frac{1}{8} \left(\frac{\bar{g}}{M_{Z'}} \right)^4 (T_{\nu_R}^{\text{dec}})^5 \sum_{i=1}^6 \mathcal{N}_i, \quad (4)$$

is the ν_R interaction rate,

$$\begin{aligned} H(T_{\nu_R}^{\text{dec}}) &= 1.66 \sqrt{g_\rho} (T_{\nu_R}^{\text{dec}})^2 / M_{\text{Pl}} \\ &\simeq 1.66 \sqrt{g_s(T_{\nu_L}^{\text{dec}})} \frac{(T_{\nu_R}^{\text{dec}})^2}{M_{\text{Pl}}} \left(\frac{3}{\Delta N_\nu} \right)^{3/8}, \end{aligned} \quad (5)$$

$\bar{g} \equiv [(\sum_{i=1}^6 \mathcal{N}_i g_i^2 g_\rho^2) / (\sum_{i=1}^6 \mathcal{N}_i)]^{1/4}$, \mathcal{N}_i is the number of chiral states, g_i are the chiral couplings of the Z' for the 6 relevant species (see below), and the constant $\mathcal{K} = 0.5$ (2.5) for annihilation (annihilation + scattering) [6]. In the second line of (5) we set $g_s \simeq g_\rho$.

The physics of interest takes place in the quark gluon plasma itself so that we will restrict ourselves to the following fermionic fields in the visible sector, $[3u_R] + [3d_R] + [3s_R] + [3\nu_L + e_L + \mu_L] + [e_R + \mu_R] + [3u_L + 3d_L + 3s_L] + [3\nu_R]$, and their contribution to g_ρ . This amounts to 28 Weyl fields, translating to 56 fermionic r.d.o.f.; $\sum_{i=1}^6 \mathcal{N}_i = 28$. To illustrate we calculate \bar{g} for two candidate models. The first is a set of variations on D-brane constructions which do not have coupling constant unification, $0.29 \leq \bar{g} \leq 0.39$ [13]. The second are $U(1)$ models which are embedded in a grand unified exceptional E_6 group, $0.09 \leq \bar{g} \leq 0.46$ [9].

Substituting (4) and (5) into (3) we obtain

$$\frac{\bar{g}}{M_{Z'}} = \left(\frac{3}{\Delta N_\nu} \right)^{3/32} \left(\frac{13.28 \sqrt{g_s(T_{\nu_L}^{\text{dec}})}}{M_{\text{Pl}} \mathcal{K} (T_{\nu_R}^{\text{dec}})^3 \sum_{i=1}^6 \mathcal{N}_i} \right)^{1/4}$$

and

$$\Delta N_\nu = \left[\frac{5.39 \times 10^{-6}}{\mathcal{H} \sum_{i=1}^6 \mathcal{N}_i} \left(\frac{M_{Z'}}{\text{TeV}} \frac{1}{\bar{g}} \right)^4 \left(\frac{\text{GeV}}{T_{\nu_R}} \right)^3 \right]^{8/3}. \quad (6)$$

In Fig. 3 we show the region of the parameter space allowed from decoupling requirements to accommodate contributions of ΔN_ν within the 1σ region of Planck data. It is important to stress that the LHC experimental limits on $M_{Z'}$ for null signals for enhancements in dilepton searches entail $M_{Z'} > 2.3$ TeV at the 95% CL [14], whereas limits from dijet final states entail $M_{Z'} > 4$ TeV at 95% CL [15].

Next, in line with our stated plan, we derive quantitative bounds on the fraction of non-thermally produced DM particles, which are relativistic at the CMB epoch. The assumption herein is that of the total DM density around today, Ω_{DM} , a small fraction, $f = \Omega_X/\Omega_{\text{DM}}$, is of particles of type X , produced via decay of a heavy relic X' with mass M' and lifetime τ : $X' \rightarrow X + \gamma$.³ At any time after the decay of X' the total DM energy density is found to be

$$\rho_{\text{DM}}(t) = \frac{M n_X(\text{today})}{a^3(t)} \gamma(t) + (1-f) \rho_c \frac{\Omega_{\text{DM}}}{a^3(t)}, \quad (7)$$

where M is the mass of the X particle, $n_X(t)$ its number density, and $a(t)$ is the expansion scale factor normalized by $a(\text{today}) = 1$. The scale factor dependence on the Lorentz boost is given by

$$\begin{aligned} \gamma(t) &= \sqrt{1 + [a(\tau)/a(t)]^2 [\gamma^2(\tau) - 1]} \\ &\approx 1 + \frac{1}{2} [a(\tau)/a(t)]^2 [\gamma^2(\tau) - 1] - \frac{1}{8} [a(\tau)/a(t)]^4 \\ &\quad \times [\gamma^2(\tau) - 1]^2 + \dots, \end{aligned} \quad (8)$$

where

$$\gamma(\tau) = \frac{M'}{2M} + \frac{M}{2M'}. \quad (9)$$

At the present day the X particles are non-relativistic, *i.e.* $\Omega_X(\text{today}) = M n_X(\text{today})/\rho_c$. To obtain such a non-relativistic limit we demand the magnitude of the second term in the expansion of (8) to be greater than the third term, which results in $[a(\tau)/a(t)]^2 [\gamma^2(\tau) - 1] < 4$. Contrariwise, by this criteria the particle X is relativistic if $\gamma(t) > \sqrt{5}$.

To obtain the X component of N_{eff} we assume that X particles decouple from the plasma prior to ν_L decoupling, conserving the ratio T_γ/T_{ν_L} from SM cosmology,

$$\begin{aligned} \Delta N_X &= \frac{8}{7} \left(\frac{11}{4} \right)^{4/3} \frac{\rho_X(t_{\text{EQ}})}{\rho_\gamma(t_{\text{EQ}})} \\ &= \frac{8}{7} \left(\frac{11}{4} \right)^{4/3} \frac{\Omega_{\text{DM}}}{\Omega_\gamma} a(t_{\text{EQ}}) \gamma(t_{\text{EQ}}) f, \end{aligned} \quad (10)$$

where $\rho_\gamma(t_{\text{EQ}}) = \rho_c \Omega_\gamma / a^4(t_{\text{EQ}})$. The radiation-matter equality time is defined by the condition $\rho_R = \rho_M$, which implies

$$\frac{1}{a(t_{\text{EQ}})} \frac{\Omega_\gamma}{\Omega_M} \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} (3 + \Delta N) \right] = 1, \quad (11)$$

with $\Delta N = \Delta N_\nu + \Delta N_X$. By expressing the scale factor in a piece-wise form,

$$a(t) = \begin{cases} \left(\frac{3}{2} H_0 t \right)^{2/3} & \text{if } t > t_{\text{EQ}} \\ \left(\frac{3}{2} H_0 t_{\text{EQ}}^{1/4} \right)^{2/3} t^{1/2} & \text{if } t_{\text{EQ}} > t > 1 \text{ s} \end{cases},$$

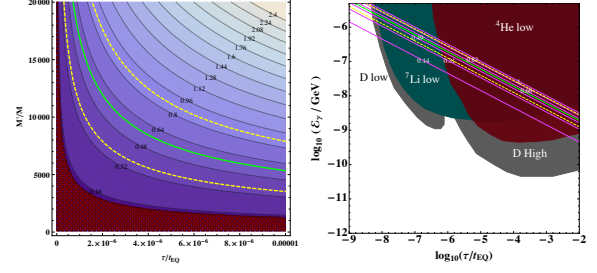


Fig. 4: Left: Contours of constant ΔN_X in the $R - \tau/t_{\text{EQ}}$ plane, with $f = 0.01$, for the case in which $\Delta N_\nu = 0$. The (green) solid line indicates the upper limit on ΔN from Planck + BAO. The band between the dashed (yellow) lines corresponds to the allowed ΔN from Planck + H_0 . The crosshatched area pertains to the region of the parameter space for which the X particles are non-relativistic at the CMB epoch, and should be therefore not taken under consideration. **Right:** The shaded regions show BBN constraints on the decay $X' \rightarrow X + \gamma$ [17]. The various contributions to ΔN_X fall on straight lines.

from (11) we obtain

$$ht_{\text{EQ}} = \frac{\text{sMpc}}{150 \text{ km}} \left\{ \frac{\Omega_\gamma}{\Omega_M} \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} (3 + \Delta N) \right] \right\}^{3/2}.$$

Note that the combination ht_{EQ} is independent of h .

At this point it is worth exploring the quantity $\gamma(t_{\text{EQ}})$ and its dependence on h ,

$$\begin{aligned} \gamma(t_{\text{EQ}}) &= \sqrt{1 + [a(\tau)/a(t_{\text{EQ}})]^2 (R^2 - 1)^2 / (4R^2)} \\ &= \sqrt{1 + (\tau/t_{\text{EQ}}) (R^2 - 1)^2 / (4R^2)}, \end{aligned} \quad (12)$$

where $R = M'/M$ and we have assumed that X' decays before the time of matter radiation equality. Interestingly, if time is scaled to t_{EQ} the analysis is h independent. This is important because of the recent tension between astrophysical data and cosmological observations. The systematics on h will seriously influence the dependence on N_{eff} . Putting all this together (10) can be rewritten as

$$\Delta N_X = \frac{\Omega_{\text{DM}}}{\Omega_M} \mathcal{A} \left[\frac{8}{7} \left(\frac{11}{4} \right)^{4/3} + 3 + \Delta N \right] f. \quad (13)$$

where

$$\mathcal{A} = \sqrt{1 + \frac{\tau}{t_{\text{EQ}}} \frac{(R^2 - 1)^2}{4R^2}}. \quad (14)$$

Note that (13) scales with the ratio $\Omega_{\text{DM}}/\Omega_M \simeq 0.84$, which does not depend on the Hubble parameter. Finally, solving (13) for ΔN_X gives

$$\Delta N_X = (7.4 + \Delta N_\nu) \mathcal{A} \left(\frac{\Omega_M}{\Omega_{\text{DM}} f} - \mathcal{A} \right)^{-1}. \quad (15)$$

In Fig. 4 we show contours of constant ΔN_X in the $R - \tau/t_{\text{EQ}}$ plane, for the case in which $\Delta N_\nu = 0$. As expected,

3. We do not at present consider the more complicated scenario in which high energy neutrinos are among the decay products [16].

to produce a given ΔN_X contribution, the required ratio of masses diminishes with increasing lifetime.

We now verify that $X' \rightarrow X + \gamma$ does not drastically alter any of the light elemental abundances synthesized during BBN. Following [17], we assume the photons injected into the plasma rapidly redistribute their energy through scattering off background photons and through inverse Compton scattering. As a consequence, the constraints from BBN are (almost) independent of the initial energy distribution of the injected photons, and are only sensitive to the total energy released in the decay process. In the spirit of [18], we conveniently write the electromagnetic energy release as $\varepsilon_\gamma \equiv E_\gamma Y_{X'}$, where $E_\gamma = (M'^2 - M^2)/(2M')$ is the initial electromagnetic energy release in each X' decay and $Y_{X'} \equiv n_{X'}/n_\gamma^{\text{BG}}$ is the number density of X' before the decay, normalized to the number density of background photons $n_\gamma^{\text{BG}} = 2 \zeta(2) T_\gamma^3/\pi^2$. For $Y_{X'}$, each X' decay produces one X , and so the X' abundance may be expressed in terms of the present X abundance through

$$Y_{X'}(\tau) = Y_{X,\tau} = Y_{X,\text{today}} = \frac{\Omega_X \rho_c}{M n_\gamma^{\text{BG}}(\text{today})} \simeq 2.26 \times 10^{-14} \frac{\text{TeV}}{M} \frac{\Omega_{\text{DM}} h^2}{0.1199} \frac{f}{0.01}, \quad (16)$$

yielding

$$\varepsilon_\gamma = 1.13 \times 10^{-11} \frac{\Omega_{\text{DM}} h^2}{0.1199} \frac{f}{0.01} \left(\frac{M'}{M} - \frac{M}{M'} \right) \text{GeV}.$$

The thorough analysis of electromagnetic cascades reported in [17] reveals that the shaded regions of Fig. 4 are ruled out by considerations of light elemental abundances produced during BBN. The various regions are disfavored by the following conservative criteria: (i) $D/H < 10^{-4.9}$ (low); (ii) $D/H > 10^{-4.3}$ (high); (iii) ${}^7\text{Li}/H < 10^{-10.05}$; (iv) primordial ${}^4\text{He}$ abundance < 0.227 . The straight lines represent several combinations of R and τ/t_{EQ} producing the ΔN_X indicated in the labels. All straight lines intersect the BBN bounds at about $\log_{10}(\tau/t_{\text{EQ}}) = -8.2$. The constraints from BBN are weak for early decays because at early times the universe is hot and thus the X' secondary photon spectrum is rapidly thermalized, leaving just a few high-energy photons that cannot alter the light elemental abundances. However, for $\tau/t_{\text{EQ}} > 10^{-8.2}$ BBN excludes most of the relevant parameter space.

Finally, we combine (6) and (15) to account for both ΔN_ν and ΔN_X components. In Fig. 5 we show contours of constant ΔN in the $M_{Z'}/(K\bar{g})$ vs. $(\tau/t_{\text{EQ}})(R^2 - 1)^2/(2R)^2$ plane. Interestingly, despite the very restrict limit from Planck + BAO data, there is room within the 1σ allowed N_{eff} region to accommodate contributions from both non-thermal DM and right-handed neutrinos which interact with all fermions via a Z' gauge boson that is within the LHC discovery reach. Note that for (say) $\Delta N_X \approx 0.3$, the particular range of DM parameters includes $(\tau/t_{\text{EQ}} \approx 10^{-8.2}$, $R \approx 6 \times 10^4$), which is not excluded by BBN considerations.

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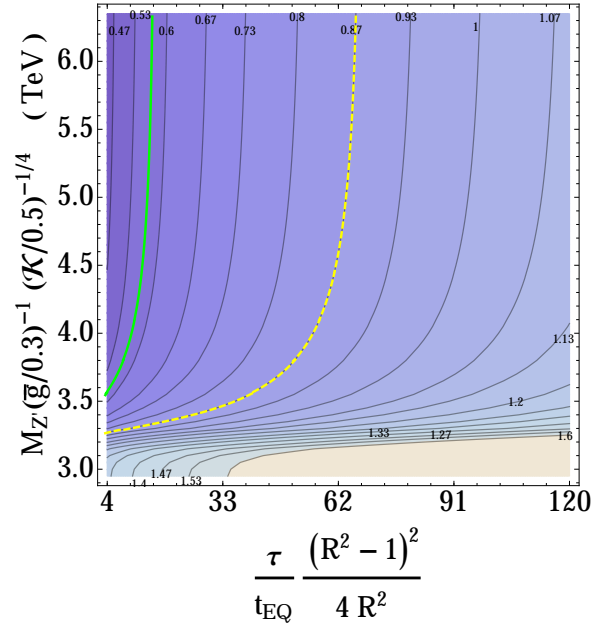


Fig. 5: Contours of constant ΔN in the $M_{Z'}/(K\bar{g})$ vs. $(\tau/t_{\text{EQ}})(R^2 - 1)^2/(2R)^2$ plane, with $f = 0.01$. The solid (dashed) line indicates the 1σ upper limit on ΔN from Planck + BAO (Planck + H_0).

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